Package 'lhs'

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Title Latin Hypercube Samples

Version 1.2.0

Description Provides a number of methods for creating and augmenting Latin Hypercube Samples and Orthogonal Array Latin Hypercube Samples.

License GPL-3

Encoding UTF-8

Depends R (>= 3.4.0)

LinkingTo Rcpp

Imports Rcpp

Suggests testthat, DoE.base, knitr, rmarkdown

URL https://github.com/bertcarnell/lhs

BugReports https://github.com/bertcarnell/lhs/issues

RoxygenNote 7.3.2

VignetteBuilder knitr

NeedsCompilation yes

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Repository CRAN

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Contents

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augmentLHS   *Augment a Latin Hypercube Design*

Description

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design.

Usage

```
augmentLHS(lhs, m = 1)
```

Arguments

lhs   The Latin Hypercube Design to which points are to be added. Contains an existing latin hypercube design with a number of rows equal to the points in the design (simulations) and a number of columns equal to the number of variables (parameters). The values of each cell must be between 0 and 1 and uniformly distributed

m   The number of additional points to add to matrix lhs

Details

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design. Augmentation is perfomed in a random manner.

The algorithm used by this function has the following steps. First, create a new matrix to hold the candidate points after the design has been re-partitioned into $(n + m)^2$ cells, where n is number of points in the original lhs matrix. Then randomly sweep through each column (1...k) in the repartitioned design to find the missing cells. For each column (variable), randomly search for an empty row, generate a random value that fits in that row, record the value in the new matrix. The new matrix can contain more filled cells than m unles $m = 2n$, in which case the new matrix will contain exactly m filled cells. Finally, keep only the first m rows of the new matrix. It is guaranteed to have m full rows in the new matrix. The deleted rows are partially full. The additional candidate points are selected randomly due to the random search for empty cells.

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

Author(s)

Rob Carnell

References

Stein, M. (1987) Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*. **29**, 143–151.

See Also

[randomLHS()], [geneticLHS()], [improvedLHS()], [maximinLHS()], and [optimumLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()] and [optSeededLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
a <- randomLHS(4,3)
b <- augmentLHS(a, 2)
```

---

correlatedLHS	*Transformed Latin hypercube with a multivariate distribution*

Description

A method to create a transformed Latin Hypercube sample where the marginal distributions can be correlated according to an arbitrary set of criteria contained in a minimized cost function

Usage

```
correlatedLHS(
  lhs,
  marginal_transform_function,
  cost_function,
  debug = FALSE,
  maxiter = 10000,
  ...
)
```

Arguments

`lhs`	a Latin hypercube sample that is uniformly distributed on the margins
`marginal_transform_function`	
	a function that takes Latin hypercube sample as the first argument and other passed-through variables as desired. `...` must be passed as a argument. For example, `f <- function(W, second_argument, ...)`. Must return a `matrix` or `data.frame`
`cost_function`	a function that takes a transformed Latin hypercube sample as the first argument and other passed-through variables as desired. `...` must be passed as a argument. For example, `f <- function(W, second_argument, ...)`
`debug`	Should debug messages be printed. Causes cost function output and iterations to be printed to aid in setting the maximum number of iterations
`maxiter`	the maximum number of iterations. The algorithm proceeds by swapping one variable of two points at a time. Each swap is an iteration.
`...`	Additional arguments to be passed through to the `marginal_transform_function` and `cost_function`

Value

a list of the Latin hypercube with uniform margins, the transformed Latin hypercube, and the final cost

Examples

```
correlatedLHS(lhs::randomLHS(30, 2),
  marginal_transform_function = function(W, ...) {
    W[,1] <- qnorm(W[,1], 1, 3)
    W[,2] <- qexp(W[,2], 2)
    return(W)
  },
  cost_function = function(W, ...) {
    (cor(W[,1], W[,2]) - 0.5)^2
  },
  debug = FALSE,
  maxiter = 1000)
```

createAddelKemp	*Create an orthogonal array using the Addelman-Kempthorne algorithm.*

Description

The addelkemp program produces OA(2q^2, k, q, 2), k <= 2q+1, for odd prime powers q.

Usage

```
createAddelKemp(q, ncol, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. https://lib.stat.cmu.edu/designs/oa.c. 1994 S. Addelman and O. Kempthorne (1961) Annals of Mathematical Statistics, Vol 32 pp 1167-1176.

See Also

Other methods to create orthogonal arrays [createBoseBush()], [createBose()], [createAddelKemp3()], [createAddelKempN()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createAddelKemp(3, 3, TRUE)
B <- createAddelKemp(3, 5, FALSE)
```

---

createAddelKemp3   *Create an orthogonal array using the Addelman-Kempthorne algorithm with* 2q^3 *rows.*

Description

The addelkemp3 program produces OA(2*q^3, k, q, 2), k <= 2q^2+2q+1, for prime powers q. q may be an odd prime power, or q may be 2 or 4.

Usage

```
createAddelKemp3(q, ncol, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. https://lib.stat.cmu.edu/designs/oa.c. 1994 S. Addelman and O. Kempthorne (1961) Annals of Mathematical Statistics, Vol 32 pp 1167-1176.

See Also

Other methods to create orthogonal arrays [createBushBush()], [createBose()], [createAddelKemp()], [createAddelKempN()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createAddelKemp3(3, 3, TRUE)
B <- createAddelKemp3(3, 5, FALSE)
```

---

createAddelKempN | *Create an orthogonal array using the Addelman-Kempthorne algorithm with alternate strength with* 2q^n *rows.*

Description

The addelkempn program produces OA(2*q^n, k, q, 2), k <= 2(q^n – 1)/(q-1)–1, for prime powers q. q may be an odd prime power, or q may be 2 or 4.

Usage

```
createAddelKempN(q, ncol, exponent, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
exponent	the exponent on q
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

See Also

Other methods to create orthogonal arrays [createBoseBush()], [createBose()], [createBush()], [createAddelKemp()], [createAddelKemp3()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createAddelKempN(3, 4, 3, TRUE)
B <- createAddelKempN(3, 4, 4, TRUE)
```

---

createBose   *Create an orthogonal array using the Bose algorithm.*

Description

The bose program produces OA(q^2, k, q, 2), k <= q+1 for prime powers q.

Usage

```
createBose(q, ncol, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. https://lib.stat.cmu.edu/designs/oa.c. 1994 R.C. Bose (1938) Sankhya Vol 3 pp 323-338

See Also

Other methods to create orthogonal arrays [createBush()], [createBoseBush()], [createAddelKemp()], [createAddelKemp3()], [createAddelKempN()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createBose(3, 3, FALSE)
B <- createBose(5, 4, TRUE)
```

---

createBoseBush *Create an orthogonal array using the Bose-Bush algorithm.*

Description

The bosebush program produces `OA( 2q^2, k, q, 2 )`, `k <= 2q+1`, for powers of 2, `q=2^r`.

Usage

```
createBoseBush(q, ncol, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
bRandom	should the array be randomized

Details

From Owen: An orthogonal array `A` is a matrix of n rows, k columns with every element being one of q symbols `0,...,q-1`. The array has strength `t` if, in every n by `t` submatrix, the `q^t` possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted `lambda`. Clearly, `lambda*q^t=n`. The notation for such an array is `OA( n, k, q, t )`.

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. <https://lib.stat.cmu.edu/designs/oa.c>. 1994 R.C. Bose and K.A. Bush (1952) Annals of Mathematical Statistics, Vol 23 pp 508-524.

See Also

Other methods to create orthogonal arrays [createBush()], [createBose()], [createAddelKemp()], [createAddelKemp3()], [createAddelKempN()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createBoseBush(4, 3, FALSE)
B <- createBoseBush(8, 3, TRUE)
```

---

createBoseBushl   *Create an orthogonal array using the Bose-Bush algorithm with alternate strength >= 3.*

Description

The bosebushl program produces OA(lambda*q^2, k, q, 2), k <= lambda*q+1, for prime powers q and lambda > 1. Both q and lambda must be powers of the same prime.

Usage

```
createBoseBushl(q, ncol, lambda, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
lambda	the lambda of the BoseBush algorithm
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. https://lib.stat.cmu.edu/designs/oa.c. 1994 R.C. Bose and K.A. Bush (1952) Annals of Mathematical Statistics, Vol 23 pp 508-524.

See Also

Other methods to create orthogonal arrays [createBoseBush()], [createBose()], [createBush()], [createAddelKemp()], [createAddelKemp3()], [createAddelKempN()], [createBusht()]

Examples

```
A <- createBoseBushl(3, 3, 3, TRUE)
B <- createBoseBushl(4, 4, 16, TRUE)
```

---

createBush   *Create an orthogonal array using the Bush algorithm.*

Description

The bush program produces OA(q^3, k, q, 3), k <= q+1 for prime powers q.

Usage

```
createBush(q, ncol, bRandom = TRUE)
```

Arguments

q	the number of symbols in the array
ncol	number of parameters or columns
bRandom	should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. <https://lib.stat.cmu.edu/designs/oa.c>. 1994 K.A. Bush (1952) Annals of Mathematical Statistics, Vol 23 pp 426-434

See Also

Other methods to create orthogonal arrays [createBoseBush()], [createBose()], [createAddelKemp()], [createAddelKemp3()], [createAddelKempN()], [createBusht()], [createBoseBushl()]

Examples

```
A <- createBush(3, 3, FALSE)
B <- createBush(4, 5, TRUE)
```

---

createBusht | *Create an orthogonal array using the Bush algorithm with alternate strength.*

Description

The busht program produces OA(q^t, k, q, t), k <= q+1, t>=3, for prime powers q.

Usage

```
createBusht(q, ncol, strength, bRandom = TRUE)
```

Arguments

q   the number of symbols in the array

ncol   number of parameters or columns

strength   the strength of the array to be created

bRandom   should the array be randomized

Details

From Owen: An orthogonal array A is a matrix of n rows, k columns with every element being one of q symbols 0,...,q-1. The array has strength t if, in every n by t submatrix, the q^t possible distinct rows, all appear the same number of times. This number is the index of the array, commonly denoted lambda. Clearly, lambda*q^t=n. The notation for such an array is OA(n, k, q, t).

Value

an orthogonal array

References

Owen, Art. Orthogonal Arrays for: Computer Experiments, Visualizations, and Integration in high dimensions. https://lib.stat.cmu.edu/designs/oa.c. 1994 K.A. Bush (1952) Annals of Mathematical Statistics, Vol 23 pp 426-434

See Also

Other methods to create orthogonal arrays [createBoseBush()], [createBose()], [createAddelKemp()], [createAddelKemp3()], [createAddelKempN()], [createBoseBushl()]

Examples

```
set.seed(1234)
A <- createBusht(3, 4, 2, TRUE)
B <- createBusht(3, 4, 3, FALSE)
G <- createBusht(3, 4, 3, TRUE)
```

`create_galois_field`   *Create a Galois field*

Description

Create a Galois field

Usage

```
create_galois_field(q)
```

Arguments

q   The order of the Galois Field q = p^n

Value

a GaloisField object containing

n q = p^n

p The prime modulus of the field q=p^n

q The order of the Galois Field q = p^n. q must be a prime power.

xton coefficients of the characteristic polynomial where the first coefficient is on x^0, the second is on x^1 and so on

inv An index for which row of `poly` (zero based) is the multiplicative inverse of this row. An `NA` indicates that this row of `poly` has no inverse. e.g. c(3, 4) means that row 4=3+1 is the inverse of row 1 and row 5=4+1 is the inverse of row 2

neg An index for which row of `poly` (zero based) is the negative or additive inverse of this row. An `NA` indicates that this row of `poly` has no negative. e.g. c(3, 4) means that row 4=3+1 is the negative of row 1 and row 5=4+1 is the negative of row 2

root An index for which row of `poly` (zero based) is the square root of this row. An `NA` indicates that this row of `poly` has no square root. e.g. c(3, 4) means that row 4=3+1 is the square root of row 1 and row 5=4+1 is the square root of row 2

plus sum table of the Galois Field

times multiplication table of the Galois Field

poly rows are polynomials of the Galois Field where the entries are the coefficients of the polynomial where the first coefficient is on x^0, the second is on x^1 and so on

Examples

```
gf <- create_galois_field(4);
```

---

create_oalhs   *Create an orthogonal array Latin hypercube*

Description

Create an orthogonal array Latin hypercube

Usage

```
create_oalhs(n, k, bChooseLargerDesign, bverbose)
```

Arguments

n	the number of samples or rows in the LHS (integer)
k	the number of parameters or columns in the LHS (integer)
bChooseLargerDesign	
	should a larger oa design be chosen than the n and k requested?
bverbose	should information be printed with execution

Value

a numeric matrix which is an orthogonal array Latin hypercube sample

Examples

```
set.seed(34)
A <- create_oalhs(9, 4, TRUE, FALSE)
B <- create_oalhs(9, 4, TRUE, FALSE)
```

---

geneticLHS   *Latin Hypercube Sampling with a Genetic Algorithm*

Description

Draws a Latin Hypercube Sample from a set of uniform distributions for use in creating a Latin Hypercube Design. This function attempts to optimize the sample with respect to the S optimality criterion through a genetic type algorithm.

Usage

```
geneticLHS(
  n = 10,
  k = 2,
  pop = 100,
  gen = 4,
  pMut = 0.1,
  criterium = "S",
  verbose = FALSE
)
```

Arguments

n	The number of partitions (simulations or design points or rows)
k	The number of replications (variables or columns)
pop	The number of designs in the initial population
gen	The number of generations over which the algorithm is applied
pMut	The probability with which a mutation occurs in a column of the progeny
criterium	The optimality criterium of the algorithm. Default is S. Maximin is also supported
verbose	Print informational messages. Default is FALSE

Details

Latin hypercube sampling (LHS) was developed to generate a distribution of collections of parameter values from a multidimensional distribution. A square grid containing possible sample points is a Latin square iff there is only one sample in each row and each column. A Latin hypercube is the generalisation of this concept to an arbitrary number of dimensions. When sampling a function of k variables, the range of each variable is divided into n equally probable intervals. n sample points are then drawn such that a Latin Hypercube is created. Latin Hypercube sampling generates more efficient estimates of desired parameters than simple Monte Carlo sampling.

This program generates a Latin Hypercube Sample by creating random permutations of the first n integers in each of k columns and then transforming those integers into n sections of a standard uniform distribution. Random values are then sampled from within each of the n sections. Once the sample is generated, the uniform sample from a column can be transformed to any distribution by using the quantile functions, e.g. qnorm(). Different columns can have different distributions.

S-optimality seeks to maximize the mean distance from each design point to all the other points in the design, so the points are as spread out as possible.

Genetic Algorithm:

1. Generate pop random latin hypercube designs of size n by k
2. Calculate the S optimality measure of each design
3. Keep the best design in the first position and throw away half of the rest of the population
4. Take a random column out of the best matrix and place it in a random column of each of the other matricies, and take a random column out of each of the other matricies and put it in copies of the best matrix thereby causing the progeny

5. For each of the progeny, cause a genetic mutation `pMut` percent of the time. The mutation is accomplished by swtching two elements in a column

Value

An `n` by `k` Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

Author(s)

Rob Carnell

References

Stocki, R. (2005) A method to improve design reliability using optimal Latin hypercube sampling *Computer Assisted Mechanics and Engineering Sciences* **12**, 87–105.

Stein, M. (1987) Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*. **29**, 143–151.

See Also

[randomLHS()], [improvedLHS()], [maximinLHS()], and [optimumLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()] [optSeededLHS()], and [augtmentLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
A <- geneticLHS(4, 3, 50, 5, .25)
```

---

get_library_versions   *Get version information for all libraries in the lhs package*

Description

Get version information for all libraries in the lhs package

Usage

```
get_library_versions()
```

Value

a character string containing the versions

Examples

```
get_library_versions()
```

---

improvedLHS   *Improved Latin Hypercube Sample*

Description

Draws a Latin Hypercube Sample from a set of uniform distributions for use in creating a Latin Hypercube Design. This function attempts to optimize the sample with respect to an optimum euclidean distance between design points.

Usage

```
improvedLHS(n, k, dup = 1)
```

Arguments

n	The number of partitions (simulations or design points or rows)
k	The number of replications (variables or columns)
dup	A factor that determines the number of candidate points used in the search. A multiple of the number of remaining points than can be added.

Details

Latin hypercube sampling (LHS) was developed to generate a distribution of collections of parameter values from a multidimensional distribution. A square grid containing possible sample points is a Latin square iff there is only one sample in each row and each column. A Latin hypercube is the generalisation of this concept to an arbitrary number of dimensions. When sampling a function of k variables, the range of each variable is divided into n equally probable intervals. n sample points are then drawn such that a Latin Hypercube is created. Latin Hypercube sampling generates more efficient estimates of desired parameters than simple Monte Carlo sampling.

This program generates a Latin Hypercube Sample by creating random permutations of the first n integers in each of k columns and then transforming those integers into n sections of a standard uniform distribution. Random values are then sampled from within each of the n sections. Once the sample is generated, the uniform sample from a column can be transformed to any distribution byusing the quantile functions, e.g. qnorm(). Different columns can have different distributions.

This function attempts to optimize the sample with respect to an optimum euclidean distance between design points.

$$Optimum distance = fracn n^{\frac{1.0}{k}}$$

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

References

Beachkofski, B., Grandhi, R. (2002) Improved Distributed Hypercube Sampling *American Institute of Aeronautics and Astronautics Paper* **1274**.

This function is based on the MATLAB program written by John Burkardt and modified 16 Feb 2005 https://people.math.sc.edu/Burkardt/m_src/ihs/ihs.html

See Also

[randomLHS()], [geneticLHS()], [maximinLHS()], and [optimumLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()], [optSeededLHS()], and [augmentLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
A <- improvedLHS(4, 3, 2)
```

---

maximinLHS   *Maximin Latin Hypercube Sample*

Description

Draws a Latin Hypercube Sample from a set of uniform distributions for use in creating a Latin Hypercube Design. This function attempts to optimize the sample by maximizing the minium distance between design points (maximin criteria).

Usage

```
maximinLHS(
  n,
  k,
  method = "build",
  dup = 1,
  eps = 0.05,
  maxIter = 100,
  optimize.on = "grid",
  debug = FALSE
)
```

Arguments

n	The number of partitions (simulations or design points or rows)
k	The number of replications (variables or columns)
method	build or iterative is the method of LHS creation. build finds the next best point while constructing the LHS. iterative optimizes the resulting sample on [0,1] or sample grid on [1,N]

dup	A factor that determines the number of candidate points used in the search. A multiple of the number of remaining points than can be added. This is used when method="build"
eps	The minimum percent change in the minimum distance used in the iterative method
maxIter	The maximum number of iterations to use in the iterative method
optimize.on	grid or result gives the basis of the optimization. grid optimizes the LHS on the underlying integer grid. result optimizes the resulting sample on [0,1]
debug	prints additional information about the process of the optimization

Details

Latin hypercube sampling (LHS) was developed to generate a distribution of collections of parameter values from a multidimensional distribution. A square grid containing possible sample points is a Latin square iff there is only one sample in each row and each column. A Latin hypercube is the generalisation of this concept to an arbitrary number of dimensions. When sampling a function of k variables, the range of each variable is divided into n equally probable intervals. n sample points are then drawn such that a Latin Hypercube is created. Latin Hypercube sampling generates more efficient estimates of desired parameters than simple Monte Carlo sampling.

This program generates a Latin Hypercube Sample by creating random permutations of the first n integers in each of k columns and then transforming those integers into n sections of a standard uniform distribution. Random values are then sampled from within each of the n sections. Once the sample is generated, the uniform sample from a column can be transformed to any distribution by using the quantile functions, e.g. qnorm(). Different columns can have different distributions.

Here, values are added to the design one by one such that the maximin criteria is satisfied.

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

References

Stein, M. (1987) Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*. **29**, 143–151.

This function is motivated by the MATLAB program written by John Burkardt and modified 16 Feb 2005 https://people.math.sc.edu/Burkardt/m_src/ihs/ihs.html

See Also

[randomLHS()], [geneticLHS()], [improvedLHS()] and [optimumLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()], [optSeededLHS()], and [augmentLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
A1 <- maximinLHS(4, 3, dup=2)
A2 <- maximinLHS(4, 3, method="build", dup=2)
```

```
A3 <- maximinLHS(4, 3, method="iterative", eps=0.05, maxIter=100, optimize.on="grid")
A4 <- maximinLHS(4, 3, method="iterative", eps=0.05, maxIter=100, optimize.on="result")
```

---

oa_to_oalhs   *Create a Latin hypercube from an orthogonal array*

Description

Create a Latin hypercube from an orthogonal array

Usage

```
oa_to_oalhs(n, k, oa)
```

Arguments

n	the number of samples or rows in the LHS (integer)
k	the number of parameters or columns in the LHS (integer)
oa	the orthogonal array to be used as the basis for the LHS (matrix of integers) or data.frame of factors

Value

a numeric matrix which is a Latin hypercube sample

Examples

```
oa <- createBose(3, 4, TRUE)
B <- oa_to_oalhs(9, 4, oa)
```

---

optAugmentLHS   *Optimal Augmented Latin Hypercube Sample*

Description

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design. This function attempts to add the points to the design in an optimal way.

Usage

```
optAugmentLHS(lhs, m = 1, mult = 2)
```

Arguments

lhs	The Latin Hypercube Design to which points are to be added
m	The number of additional points to add to matrix lhs
mult	m*mult random candidate points will be created.

Details

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design. This function attempts to add the points to the design in a way that maximizes S optimality.

S-optimality seeks to maximize the mean distance from each design point to all the other points in the design, so the points are as spread out as possible.

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

References

Stein, M. (1987) Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*. **29**, 143–151.

See Also

[randomLHS()], [geneticLHS()], [improvedLHS()], [maximinLHS()], and [optimumLHS()] to generate Latin Hypercube Samples. [optSeededLHS()] and [augmentLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
a <- randomLHS(4,3)
b <- optAugmentLHS(a, 2, 3)
```

---

optimumLHS *Optimum Latin Hypercube Sample*

Description

Draws a Latin Hypercube Sample from a set of uniform distributions for use in creating a Latin Hypercube Design. This function uses the Columnwise Pairwise (CP) algorithm to generate an optimal design with respect to the S optimality criterion.

Usage

```
optimumLHS(n = 10, k = 2, maxSweeps = 2, eps = 0.1, verbose = FALSE)
```

Arguments

n	The number of partitions (simulations or design points or rows)
k	The number of replications (variables or columns)
maxSweeps	The maximum number of times the CP algorithm is applied to all the columns.
eps	The optimal stopping criterion. Algorithm stops when the change in optimality measure is less than eps*100% of the previous value.
verbose	Print informational messages

Details

Latin hypercube sampling (LHS) was developed to generate a distribution of collections of parameter values from a multidimensional distribution. A square grid containing possible sample points is a Latin square iff there is only one sample in each row and each column. A Latin hypercube is the generalisation of this concept to an arbitrary number of dimensions. When sampling a function of k variables, the range of each variable is divided into n equally probable intervals. n sample points are then drawn such that a Latin Hypercube is created. Latin Hypercube sampling generates more efficient estimates of desired parameters than simple Monte Carlo sampling.

This program generates a Latin Hypercube Sample by creating random permutations of the first n integers in each of k columns and then transforming those integers into n sections of a standard uniform distribution. Random values are then sampled from within each of the n sections. Once the sample is generated, the uniform sample from a column can be transformed to any distribution by using the quantile functions, e.g. qnorm(). Different columns can have different distributions.

S-optimality seeks to maximize the mean distance from each design point to all the other points in the design, so the points are as spread out as possible.

This function uses the CP algorithm to generate an optimal design with respect to the S optimality criterion.

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

References

Stocki, R. (2005) A method to improve design reliability using optimal Latin hypercube sampling *Computer Assisted Mechanics and Engineering Sciences* **12**, 87–105.

See Also

[randomLHS()], [geneticLHS()], [improvedLHS()] and [maximinLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()], [optSeededLHS()], and [augmentLHS()] to modify and augment existing designs.

Examples

```
A <- optimumLHS(4, 3, 5, .05)
```

---

optSeededLHS *Optimum Seeded Latin Hypercube Sample*

Description

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design. This function then uses the columnwise pairwise (CP) algoritm to optimize the design. The original design is not necessarily maintained.

Usage

```
optSeededLHS(seed, m = 0, maxSweeps = 2, eps = 0.1, verbose = FALSE)
```

Arguments

seed	The number of partitions (simulations or design points)
m	The number of additional points to add to the seed matrix seed. default value is zero. If m is zero then the seed design is optimized.
maxSweeps	The maximum number of times the CP algorithm is applied to all the columns.
eps	The optimal stopping criterion
verbose	Print informational messages

Details

Augments an existing Latin Hypercube Sample, adding points to the design, while maintaining the *latin* properties of the design. This function then uses the CP algoritm to optimize the design. The original design is not necessarily maintained.

Value

An n by k Latin Hypercube Sample matrix with values uniformly distributed on [0,1]

References

Stein, M. (1987) Large Sample Properties of Simulations Using Latin Hypercube Sampling. *Technometrics*. **29**, 143–151.

See Also

[randomLHS()], [geneticLHS()], [improvedLHS()], [maximinLHS()], and [optimumLHS()] to generate Latin Hypercube Samples. [optAugmentLHS()] and [augmentLHS()] to modify and augment existing designs.

Examples

```
set.seed(1234)
a <- randomLHS(4,3)
b <- optSeededLHS(a, 2, 2, .1)
```

---

poly2int *Convert polynomial to integer in <code>0..q-1</code>*

Description

Convert polynomial to integer in <code>0..q-1</code>

Usage

```
poly2int(p, n, poly)
```

Arguments

p	modulus
n	the length of poly
poly	the polynomial vector

Value

an integer

Examples

```
gf <- create_galois_field(4)
stopifnot(poly2int(gf$p, gf$n, c(0, 0)) == 0)
```

---

poly_prod *Multiplication in polynomial representation*

Description

Multiplication in polynomial representation

Usage

```
poly_prod(p, n, xton, p1, p2)
```

Arguments

p	modulus
n	length of polynomials
xton	characteristic polynomial vector for the field (x to the n power)
p1	polynomial vector 1
p2	polynomial vector 2

Value

the product of p1 and p2

Examples

```
gf <- create_galois_field(4)
a <- poly_prod(gf$p, gf$n, gf$xton, c(1, 0), c(0, 1))
stopifnot(all(a == c(0, 1)))
```

---

poly_sum   *Addition in polynomial representation*

Description

Addition in polynomial representation

Usage

```
poly_sum(p, n, p1, p2)
```

Arguments

p	modulus
n	length of polynomial 1 and 2
p1	polynomial vector 1
p2	polynomial vector 2

Value

the sum of p1 and p2

Examples

```
gf <- create_galois_field(4)
a <- poly_sum(gf$p, gf$n, c(1, 0), c(0, 1))
stopifnot(all(a == c(1, 1)))
```

---

qfactor   *Quantile Transformations*

Description

A collection of functions that transform the margins of a Latin hypercube sample in multiple ways

Usage

```
qfactor(p, fact)

qinteger(p, a, b)

qdirichlet(X, alpha)
```

Arguments

p	a vector of LHS samples on (0,1)
fact	a factor or categorical variable. Ordered and un-ordered variables are allowed.
a	a minimum integer
b	a maximum integer
X	multiple columns of an LHS sample on (0,1)
alpha	Dirichlet distribution parameters. All `alpha >= 1` The marginal mean probability of the Dirichlet distribution is given by `alpha[i] / sum(alpha)`

Details

`qdirichlet` is not an exact quantile function since the quantile of a multivariate distribution is not unique. `qdirichlet` is also not the independent quantiles of the marginal distributions since those quantiles do not sum to one. `qdirichlet` is the quantile of the underlying gamma functions, normalized. This is the same procedure that is used to generate random deviates from the Dirichlet distribution therefore it will produce transformed Latin hypercube samples with the intended distribution.

`q_factor` divides the [0,1] interval into `nlevel(fact)` equal sections and assigns values in those sections to the factor level.

Value

the transformed column or columns

Examples

```
X <- randomLHS(20, 7)
Y <- as.data.frame(X)
Y[,1] <- qnorm(X[,1], 2, 0.5)
Y[,2] <- qfactor(X[,2], factor(LETTERS[c(1,3,5,7,8)]))
Y[,3] <- qinteger(X[,3], 5, 17)
Y[,4:6] <- qdirichlet(X[,4:6], c(2,3,4))
Y[,7] <- qfactor(X[,7], ordered(LETTERS[c(1,3,5,7,8)]))
```

---

randomLHS	*Construct a random Latin hypercube design*

Description

`randomLHS(4,3)` returns a 4x3 matrix with each column constructed as follows: A random permutation of (1,2,3,4) is generated, say (3,1,2,4) for each of K columns. Then a uniform random number is picked from each indicated quartile. In this example a random number between .5 and .75 is chosen, then one between 0 and .25, then one between .25 and .5, finally one between .75 and 1.

Usage

```
randomLHS(n, k, preserveDraw = FALSE)
```

Arguments

n	the number of rows or samples
k	the number of columns or parameters/variables
preserveDraw	should the draw be constructed so that it is the same for variable numbers of columns?

Value

a Latin hypercube sample

Examples

```
a <- randomLHS(5, 3)
```

---

runifint   *Create a Random Sample of Uniform Integers*

Description

Create a Random Sample of Uniform Integers

Usage

```
runifint(n = 1, min_int = 0, max_int = 1)
```

Arguments

n	The number of samples
min_int	the minimum integer x >= min_int
max_int	the maximum integer x <= max_int

Value

the sample sample of size n

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