# Package 'markovchain'

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**Type** Package

**Title** Easy Handling Discrete Time Markov Chains

**Version** 0.10.0

**Maintainer** Giorgio Alfredo Spedicato <spedicato_giorgio@yahoo.it>

**Description** Functions and S4 methods to create and manage discrete time Markov chains more easily. In addition functions to perform statistical (fitting and drawing random variates) and probabilistic (analysis of their structural proprieties) analysis are provided. See Spedicato (2017) <doi:10.32614/RJ-2017-036>. Some functions for continuous times Markov chains depend on the suggested ctmcd package.

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**Depends** R (>= 4.2.0), Matrix (>= 1.5-0), methods

**Imports** igraph, expm, stats4, parallel, Rcpp (>= 1.0.2), RcppParallel, utils, stats, grDevices

**Suggests** knitr, testthat, diagram, DiagrammeR, msm, Rsolnp, rmarkdown, ctmcd, bookdown, rticles

**Enhances** etm

**VignetteBuilder** rmarkdown, knitr, bookdown, rticles

**LinkingTo** Rcpp, RcppParallel, RcppArmadillo (>= 0.9.600.4.0)

**SystemRequirements** GNU make

**LazyLoad** yes

**ByteCompile** yes

**Encoding** UTF-8

**BugReports** https://github.com/spedygiorgio/markovchain/issues

**URL** https://github.com/spedygiorgio/markovchain/

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**NeedsCompilation** yes

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# Contents

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`markovchain-package` *Easy Handling Discrete Time Markov Chains*

---

## Description

The package contains classes and method to create and manage (plot, print, export for example) discrete time Markov chains (DTMC). In addition it provide functions to perform statistical (fitting and drawing random variates) and probabilistic (analysis of DTMC proprieties) analysis

## Author(s)

Giorgio Alfredo Spedicato Maintainer: Giorgio Alfredo Spedicato <spedicato_giorgio@yahoo.it>

## References

Discrete-Time Markov Models, Bremaud, Springer 1999

## See Also

Useful links:

- https://github.com/spedygiorgio/markovchain/
- Report bugs at https://github.com/spedygiorgio/markovchain/issues

## Examples

```
# create some markov chains
statesNames=c("a","b")
mcA<-new("markovchain", transitionMatrix=matrix(c(0.7,0.3,0.1,0.9),byrow=TRUE,
         nrow=2, dimnames=list(statesNames,statesNames)))

statesNames=c("a","b","c")
mcB<-new("markovchain", states=statesNames, transitionMatrix=
         matrix(c(0.2,0.5,0.3,0,1,0,0.1,0.8,0.1), nrow=3,
         byrow=TRUE, dimnames=list(statesNames, statesNames)))

statesNames=c("a","b","c","d")
matrice<-matrix(c(0.25,0.75,0,0,0.4,0.6,0,0,0,0,0.1,0.9,0,0,0.7,0.3), nrow=4, byrow=TRUE)
mcC<-new("markovchain", states=statesNames, transitionMatrix=matrice)
mcD<-new("markovchain", transitionMatrix=matrix(c(0,1,0,1), nrow=2,byrow=TRUE))


#operations with S4 methods
mcA^2
steadyStates(mcB)
absorbingStates(mcB)
markovchainSequence(n=20, markovchain=mcC, include=TRUE)
```

---

# absorptionProbabilities

*Absorption probabilities*

---

## Description

Computes the absorption probability from each transient state to each recurrent one (i.e. the (i, j) entry or (j, i), in a stochastic matrix by columns, represents the probability that the first not transient state we can go from the transient state i is j (and therefore we are going to be absorbed in the communicating recurrent class of j)

## Usage

```
absorptionProbabilities(object)
```

### Arguments

| | |
|---|---|
| `object` | the markovchain object |

### Value

A named vector with the expected number of steps to go from a transient state to any of the recurrent ones

### Author(s)

Ignacio Cordón

### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

### Examples

```
m <- matrix(c(1/2, 1/2, 0,
              1/2, 1/2, 0,
                0, 1/2, 1/2), ncol = 3, byrow = TRUE)
mc <- new("markovchain", states = letters[1:3], transitionMatrix = m)
absorptionProbabilities(mc)
```

---

## `blanden` *Mobility between income quartiles*

---

### Description

This table show mobility between income quartiles for father and sons for the 1970 cohort born

### Usage

```
data(blanden)
```

### Format

An object of class `table` with 4 rows and 4 columns.

### Details

The rows represent fathers' income quartile when the son is aged 16, whilst the columns represent sons' income quartiles when he is aged 30 (in 2000).

### Source

Personal reworking

## References

Jo Blanden, Paul Gregg and Stephen Machin, Intergenerational Mobility in Europe and North America, Center for Economic Performances (2005)

## Examples

```
data(blanden)
mobilityMc<-as(blanden, "markovchain")
```

---

`committorAB` *Calculates committor of a markovchain object with respect to set A, B*

---

## Description

Returns the probability of hitting states rom set A before set B with different initial states

## Usage

```
committorAB(object,A,B,p)
```

## Arguments

| | |
|---|---|
| `object` | a markovchain class object |
| `A` | a set of states |
| `B` | a set of states |
| `p` | initial state (default value : 1) |

## Details

The function solves a system of linear equations to calculate probaility that the process hits a state from set A before any state from set B

## Value

Return a vector of probabilities in case initial state is not provided else returns a number

## Examples

```
transMatr <- matrix(c(0,0,0,1,0.5,
                      0.5,0,0,0,0,
                      0.5,0,0,0,0,
                      0,0.2,0.4,0,0,
                      0,0.8,0.6,0,0.5),
                      nrow = 5)
object <- new("markovchain", states=c("a","b","c","d","e"),transitionMatrix=transMatr)
committorAB(object,c(5),c(3))
```

conditionalDistribution conditionalDistribution *of a Markov Chain*

## Description

It extracts the conditional distribution of the subsequent state, given current state.

## Usage

```
conditionalDistribution(object, state)
```

## Arguments

| | |
|---|---|
| object | A markovchain object. |
| state | Subsequent state. |

## Value

A named probability vector

## Author(s)

Giorgio Spedicato, Deepak Yadav

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchain

## Examples

```
# define a markov chain
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
               matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1),nrow = 3,
                      byrow = TRUE, dimnames = list(statesNames, statesNames)))

conditionalDistribution(markovB, "b")
```

## craigsendi *CD4 cells counts on HIV Infects between zero and six month*

### Description

This is the table shown in Craig and Sendi paper showing zero and six month CD4 cells count in six brakets

### Usage

```
data(craigsendi)
```

### Format

The format is: table [1:3, 1:3] 682 154 19 33 64 19 25 47 43 - attr(*, "dimnames")=List of 2 ..$ : chr [1:3] "0-49" "50-74" "75-UP" ..$ : chr [1:3] "0-49" "50-74" "75-UP"

### Details

Rows represent counts at the beginning, cols represent counts after six months.

### Source

Estimation of the transition matrix of a discrete time Markov chain, Bruce A. Craig and Peter P. Sendi, Health Economics 11, 2002.

### References

see source

### Examples

```
data(craigsendi)
csMc<-as(craigsendi, "markovchain")
steadyStates(csMc)
```

## createSequenceMatrix *Function to fit a discrete Markov chain*

### Description

Given a sequence of states arising from a stationary state, it fits the underlying Markov chain distribution using either MLE (also using a Laplacian smoother), bootstrap or by MAP (Bayesian) inference.

## Usage

```
createSequenceMatrix(
  stringchar,
  toRowProbs = FALSE,
  sanitize = FALSE,
  possibleStates = character()
)

markovchainFit(
  data,
  method = "mle",
  byrow = TRUE,
  nboot = 10L,
  laplacian = 0,
  name = "",
  parallel = FALSE,
  confidencelevel = 0.95,
  confint = TRUE,
  hyperparam = matrix(),
  sanitize = FALSE,
  possibleStates = character()
)
```

## Arguments

| | |
|---|---|
| stringchar | It can be a $nxn$ matrix or a character vector or a list |
| toRowProbs | converts a sequence matrix into a probability matrix |
| sanitize | put 1 in all rows having rowSum equal to zero |
| possibleStates | Possible states which are not present in the given sequence |
| data | It can be a character vector or a $nxn$ matrix or a $nxn$ data frame or a list |
| method | Method used to estimate the Markov chain. Either "mle", "map", "bootstrap" or "laplace" |
| byrow | it tells whether the output Markov chain should show the transition probabilities by row. |
| nboot | Number of bootstrap replicates in case "bootstrap" is used. |
| laplacian | Laplacian smoothing parameter, default zero. It is only used when "laplace" method is chosen. |
| name | Optional character for name slot. |

`parallel` Use parallel processing when performing Boostrap estimates.

`confidencelevel`

$$\alpha$$

level for conficence intervals width. Used only when `method` equal to "mle".

`confint` a boolean to decide whether to compute Confidence Interval or not.

`hyperparam` Hyperparameter matrix for the a priori distribution. If none is provided, default value of 1 is assigned to each parameter. This must be of size

$$kxk$$

where k is the number of states in the chain and the values should typically be non-negative integers.

## Details

Disabling confint would lower the computation time on large datasets. If `data` or `stringchar` contain `NAs`, the related `NA` containing transitions will be ignored.

## Value

A list containing an estimate, log-likelihood, and, when "bootstrap" method is used, a matrix of standards deviations and the bootstrap samples. When the "mle", "bootstrap" or "map" method is used, the lower and upper confidence bounds are returned along with the standard error. The "map" method also returns the expected value of the parameters with respect to the posterior distribution.

## Note

This function has been rewritten in Rcpp. Bootstrap algorithm has been defined "heuristically". In addition, parallel facility is not complete, involving only a part of the bootstrap process. When `data` is either a `data.frame` or a `matrix` object, only MLE fit is currently available.

## Author(s)

Giorgio Spedicato, Tae Seung Kang, Sai Bhargav Yalamanchi

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

Inferring Markov Chains: Bayesian Estimation, Model Comparison, Entropy Rate, and Out-of-Class Modeling, Christopher C. Strelioff, James P. Crutchfield, Alfred Hubler, Santa Fe Institute

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

## See Also

`markovchainSequence`, `markovchainListFit`

## Examples

```
sequence <- c("a", "b", "a", "a", "a", "a", "b", "a", "b", "a", "b", "a", "a",
              "b", "b", "b", "a")
sequenceMatr <- createSequenceMatrix(sequence, sanitize = FALSE)
mcFitMLE <- markovchainFit(data = sequence)
mcFitBSP <- markovchainFit(data = sequence, method = "bootstrap", nboot = 5, name = "Bootstrap Mc")

na.sequence <- c("a", NA, "a", "b")
# There will be only a (a,b) transition
na.sequenceMatr <- createSequenceMatrix(na.sequence, sanitize = FALSE)
mcFitMLE <- markovchainFit(data = na.sequence)

# data can be a list of character vectors
sequences <- list(x = c("a", "b", "a"), y = c("b", "a", "b", "a", "c"))
mcFitMap <- markovchainFit(sequences, method = "map")
mcFitMle <- markovchainFit(sequences, method = "mle")
```

---

# ctmc-class *Continuous time Markov Chains class*

---

## Description

The S4 class that describes `ctmc` (continuous time Markov chain) objects.

## Arguments

| | |
|---|---|
| `states` | Name of the states. Must be the same of `colnames` and `rownames` of the generator matrix |
| `byrow` | TRUE or FALSE. Indicates whether the given matrix is stochastic by rows or by columns |
| `generator` | Square generator matrix |
| `name` | Optional character name of the Markov chain |

## Methods

**dim** `signature(x = "ctmc")`: method to get the size

**initialize** `signature(.Object = "ctmc")`: initialize method

**states** `signature(object = "ctmc")`: states method.

**steadyStates** `signature(object = "ctmc")`: method to get the steady state vector.

**plot** `signature(x = "ctmc", y = "missing")`: plot method for `ctmc` objects

## Note

1. `ctmc` classes are written using S4 classes
2. Validation method is used to assess whether either columns or rows totals to zero. Rounding is used up to 5th decimal. If state names are not properly defined for a generator `matrix`, coercing to `ctmc` object leads to overriding states name with artificial "s1", "s2", ... sequence

## References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison. Sai Bhargav Yalamanchi, Giorgio Spedicato

## See Also

generatorToTransitionMatrix,rctmc

## Examples

```
energyStates <- c("sigma", "sigma_star")
byRow <- TRUE
gen <- matrix(data = c(-3, 3,
                       1, -1), nrow = 2,
              byrow = byRow, dimnames = list(energyStates, energyStates))
molecularCTMC <- new("ctmc", states = energyStates,
                     byrow = byRow, generator = gen,
                     name = "Molecular Transition Model")
                     steadyStates(molecularCTMC)
## Not run: plot(molecularCTMC)
```

---

# ctmcFit *Function to fit a CTMC*

---

## Description

This function fits the underlying CTMC give the state transition data and the transition times using the maximum likelihood method (MLE)

## Usage

```
ctmcFit(data, byrow = TRUE, name = "", confidencelevel = 0.95)
```

## Arguments

| | |
|---|---|
| data | It is a list of two elements. The first element is a character vector denoting the states. The second is a numeric vector denoting the corresponding transition times. |
| byrow | Determines if the output transition probabilities of the underlying embedded DTMC are by row. |
| name | Optional name for the CTMC. |
| confidencelevel | Confidence level for the confidence interval construction. |

### Details

Note that in data, there must exist an element wise corresponding between the two elements of the list and that data[[2]][1] is always 0.

### Value

It returns a list containing the CTMC object and the confidence intervals.

### Author(s)

Sai Bhargav Yalamanchi

### References

Continuous Time Markov Chains (vignette), Sai Bhargav Yalamanchi, Giorgio Alfredo Spedicato 2015

### See Also

rctmc

### Examples

```
data <- list(c("a", "b", "c", "a", "b", "a", "c", "b", "c"), c(0, 0.8, 2.1, 2.4, 4, 5, 5.9, 8.2, 9))
ctmcFit(data)
```

---

## expectedRewards *Expected Rewards for a markovchain*

---

### Description

Given a markovchain object and reward values for every state, function calculates expected reward value after n steps.

### Usage

```
expectedRewards(markovchain,n,rewards)
```

### Arguments

| | |
|---|---|
| markovchain | the markovchain-class object |
| n | no of steps of the process |
| rewards | vector depicting rewards coressponding to states |

## Details

the function uses a dynamic programming approach to solve a recursive equation described in reference.

## Value

returns a vector of expected rewards for different initial states

## Author(s)

Vandit Jain

## References

Stochastic Processes: Theory for Applications, Robert G. Gallager, Cambridge University Press

## Examples

```
transMatr<-matrix(c(0.99,0.01,0.01,0.99),nrow=2,byrow=TRUE)
simpleMc<-new("markovchain", states=c("a","b"),
              transitionMatrix=transMatr)
expectedRewards(simpleMc,1,c(0,1))
```

---

# expectedRewardsBeforeHittingA

*Expected first passage Rewards for a set of states in a markovchain*

---

## Description

Given a markovchain object and reward values for every state, function calculates expected reward value for a set A of states after n steps.

## Usage

```
expectedRewardsBeforeHittingA(markovchain, A, state, rewards, n)
```

## Arguments

| | |
|---|---|
| markovchain | the markovchain-class object |
| A | set of states for first passage expected reward |
| state | initial state |
| rewards | vector depicting rewards coressponding to states |
| n | no of steps of the process |

## Details

The function returns the value of expected first passage rewards given rewards coressponding to every state, an initial state and number of steps.

### Value

returns a expected reward (numerical value) as described above

### Author(s)

Sai Bhargav Yalamanchi, Vandit Jain

---

`ExpectedTime` *Returns expected hitting time from state i to state j*

---

### Description

Returns expected hitting time from state i to state j

### Usage

```
ExpectedTime(C,i,j,useRCpp)
```

### Arguments

| | |
|---|---|
| `C` | A CTMC S4 object |
| `i` | Initial state i |
| `j` | Final state j |
| `useRCpp` | logical whether to use Rcpp |

### Details

According to the theorem, holding times for all states except j should be greater than 0.

### Value

A numerical value that returns expected hitting times from i to j

### Author(s)

Vandit Jain

### References

Markovchains, J. R. Norris, Cambridge University Press

## Examples

```
states <- c("a","b","c","d")
byRow <- TRUE
gen <- matrix(data = c(-1, 1/2, 1/2, 0, 1/4, -1/2, 0, 1/4, 1/6, 0, -1/3, 1/6, 0, 0, 0, 0),
nrow = 4,byrow = byRow, dimnames = list(states,states))
ctmc <- new("ctmc",states = states, byrow = byRow, generator = gen, name = "testctmc")
ExpectedTime(ctmc,1,4,TRUE)
```

---

`firstPassage` *First passage across states*

---

## Description

This function compute the first passage probability in states

## Usage

```
firstPassage(object, state, n)
```

## Arguments

| | |
|---|---|
| `object` | A `markovchain` object |
| `state` | Initial state |
| `n` | Number of rows on which compute the distribution |

## Details

Based on Feres' Matlab listings

## Value

A matrix of size 1:n x number of states showing the probability of the first time of passage in states to be exactly the number in the row.

## Author(s)

Giorgio Spedicato

## References

Renaldo Feres, Notes for Math 450 Matlab listings for Markov chains

## See Also

`conditionalDistribution`

### Examples

```
simpleMc <- new("markovchain", states = c("a", "b"),
                transitionMatrix = matrix(c(0.4, 0.6, .3, .7),
                                          nrow = 2, byrow = TRUE))
firstPassage(simpleMc, "b", 20)
```

---

## firstPassageMultiple *function to calculate first passage probabilities*

---

### Description

The function calculates first passage probability for a subset of states given an initial state.

### Usage

```
firstPassageMultiple(object, state, set, n)
```

### Arguments

| | |
|---|---|
| object | a markovchain-class object |
| state | intital state of the process (charactervector) |
| set | set of states A, first passage of which is to be calculated |
| n | Number of rows on which compute the distribution |

### Value

A vector of size n showing the first time proabilities

### Author(s)

Vandit Jain

### References

Renaldo Feres, Notes for Math 450 Matlab listings for Markov chains; MIT OCW, course - 6.262, Discrete Stochastic Processes, course-notes, chap -05

### See Also

firstPassage

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
matrix(c(0.2, 0.5, 0.3,
         0, 1, 0,
         0.1, 0.8, 0.1), nrow = 3, byrow = TRUE,
       dimnames = list(statesNames, statesNames)
))
firstPassageMultiple(markovB,"a",c("b","c"),4)
```

---

`fitHigherOrder` *Functions to fit a higher order Markov chain*

---

## Description

Given a sequence of states arising from a stationary state, it fits the underlying Markov chain distribution with higher order.

## Usage

```
fitHigherOrder(sequence, order = 2)
seq2freqProb(sequence)
seq2matHigh(sequence, order)
```

## Arguments

| | |
|---|---|
| `sequence` | A character list. |
| `order` | Markov chain order |

## Value

A list containing lambda, Q, and X.

## Author(s)

Giorgio Spedicato, Tae Seung Kang

## References

Ching, W. K., Huang, X., Ng, M. K., & Siu, T. K. (2013). Higher-order markov chains. In Markov Chains (pp. 141-176). Springer US.

Ching, W. K., Ng, M. K., & Fung, E. S. (2008). Higher-order multivariate Markov chains and their applications. Linear Algebra and its Applications, 428(2), 492-507.

## Examples

```
sequence<-c("a", "a", "b", "b", "a", "c", "b", "a", "b", "c", "a", "b",
            "c", "a", "b", "c", "a", "b", "a", "b")
fitHigherOrder(sequence)
```

---

`fitHighOrderMultivarMC` *Function to fit Higher Order Multivariate Markov chain*

---

## Description

Given a matrix of categorical sequences it fits Higher Order Multivariate Markov chain.

## Usage

```
fitHighOrderMultivarMC(seqMat, order = 2, Norm = 2)
```

## Arguments

| | |
|---|---|
| `seqMat` | a matrix or a data frame where each column is a categorical sequence |
| `order` | Multivariate Markov chain order. Default is 2. |
| `Norm` | Norm to be used. Default is 2. |

## Value

an hommc object

## Author(s)

Giorgio Spedicato, Deepak Yadav

## References

W.-K. Ching et al. / Linear Algebra and its Applications

## Examples

```
data <- matrix(c('2', '1', '3', '3', '4', '3', '2', '1', '3', '3', '2', '1',
                 c('2', '4', '4', '4', '4', '2', '3', '3', '1', '4', '3', '3')),
                 ncol = 2, byrow = FALSE)

fitHighOrderMultivarMC(data, order = 2, Norm = 2)
```

`freq2Generator` *Returns a generator matrix corresponding to frequency matrix*

## Description

The function provides interface to calculate generator matrix corresponding to a frequency matrix and time taken

## Usage

```
freq2Generator(P, t = 1, method = "QO", logmethod = "Eigen")
```

## Arguments

| | |
|---|---|
| `P` | relative frequency matrix |
| `t` | (default value = 1) |
| `method` | one among "QO"(Quasi optimaisation), "WA"(weighted adjustment), "DA"(diagonal adjustment) |
| `logmethod` | method for computation of matrx algorithm (by default : Eigen) |

## Value

returns a generator matix with same dimnames

## References

E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. Algo Research Quarterly 4(1):23-40, 2001

## Examples

```
sample <- matrix(c(150,2,1,1,1,200,2,1,2,1,175,1,1,1,1,150),nrow = 4,byrow = TRUE)
sample_rel = rbind((sample/rowSums(sample))[1:dim(sample)[1]-1,],c(rep(0,dim(sample)[1]-1),1))
freq2Generator(sample_rel,1)

data(tm_abs)
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))
## Derive quasi optimization generator matrix estimate
freq2Generator(tm_rel,1)
```

`generatorToTransitionMatrix` *Function to obtain the transition matrix from the generator*

## Description

The transition matrix of the embedded DTMC is inferred from the CTMC's generator

## Usage

```
generatorToTransitionMatrix(gen, byrow = TRUE)
```

## Arguments

| | |
|---|---|
| `gen` | The generator matrix |
| `byrow` | Flag to determine if rows (columns) sum to 0 |

## Value

Returns the transition matrix.

## Author(s)

Sai Bhargav Yalamanchi

## References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison

## See Also

`rctmc`,`ctmc-class`

## Examples

```
energyStates <- c("sigma", "sigma_star")
byRow <- TRUE
gen <- matrix(data = c(-3, 3, 1, -1), nrow = 2,
              byrow = byRow, dimnames = list(energyStates, energyStates))
generatorToTransitionMatrix(gen)
```

## HigherOrderMarkovChain-class *Higher order Markov Chains class*

### Description

The S4 class that describes HigherOrderMarkovChain objects.

## hittingProbabilities *Hitting probabilities for markovchain*

### Description

Given a markovchain object, this function calculates the probability of ever arriving from state i to j

### Usage

```
hittingProbabilities(object)
```

### Arguments

| | |
|---|---|
| object | the markovchain-class object |

### Value

a matrix of hitting probabilities

### Author(s)

Ignacio Cordón

### References

R. Vélez, T. Prieto, Procesos Estocásticos, Librería UNED, 2013

### Examples

```
M <- markovchain:::zeros(5)
M[1,1] <- M[5,5] <- 1
M[2,1] <- M[2,3] <- 1/2
M[3,2] <- M[3,4] <- 1/2
M[4,2] <- M[4,5] <- 1/2

mc <- new("markovchain", transitionMatrix = M)
hittingProbabilities(mc)
```

holson *Holson data set*

## Description

A data set containing 1000 life histories trajectories and a categorical status (1,2,3) observed on eleven evenly spaced steps.

## Usage

```
data(holson)
```

## Format

A data frame with 1000 observations on the following 12 variables.

`id` unique id

`time1` observed status at i-th time

`time2` observed status at i-th time

`time3` observed status at i-th time

`time4` observed status at i-th time

`time5` observed status at i-th time

`time6` observed status at i-th time

`time7` observed status at i-th time

`time8` observed status at i-th time

`time9` observed status at i-th time

`time10` observed status at i-th time

`time11` observed status at i-th time

## Details

The example can be used to fit a `markovchain` or a `markovchainList` object.

## Source

Private communications

## References

Private communications

## Examples

```
data(holson)
head(holson)
```

## hommc-class — *An S4 class for representing High Order Multivariate Markovchain (HOMMC)*

### Description

An S4 class for representing High Order Multivariate Markovchain (HOMMC)

### Slots

`order` an integer equal to order of Multivariate Markovchain

`states` a vector of states present in the HOMMC model

`P` array of transition matrices

`Lambda` a vector which stores the weightage of each transition matrices in P

`byrow` if FALSE each column sum of transition matrix is 1 else row sum = 1

`name` a name given to hommc

### Author(s)

Giorgio Spedicato, Deepak Yadav

### Examples

```
statesName <- c("a", "b")

P <- array(0, dim = c(2, 2, 4), dimnames = list(statesName, statesName))
P[,,1] <- matrix(c(0, 1, 1/3, 2/3), byrow = FALSE, nrow = 2)
P[,,2] <- matrix(c(1/4, 3/4, 0, 1), byrow = FALSE, nrow = 2)
P[,,3] <- matrix(c(1, 0, 1/3, 2/3), byrow = FALSE, nrow = 2)
P[,,4] <- matrix(c(3/4, 1/4, 0, 1), byrow = FALSE, nrow = 2)

Lambda <- c(0.8, 0.2, 0.3, 0.7)

ob <- new("hommc", order = 1, states = statesName, P = P,
          Lambda = Lambda, byrow = FALSE, name = "FOMMC")
```

## ictmc-class — *An S4 class for representing Imprecise Continuous Time Markovchains*

### Description

An S4 class for representing Imprecise Continuous Time Markovchains

### Slots

`states` a vector of states present in the ICTMC model

`Q` matrix representing the generator demonstrated in the form of variables

`range` a matrix that stores values of range of variables

`name` name given to ICTMC

---

`impreciseProbabilityatT` *Calculating full conditional probability using lower rate transition matrix*

---

### Description

This function calculates full conditional probability at given time s using lower rate transition matrix

### Usage

```
impreciseProbabilityatT(C,i,t,s,error,useRCpp)
```

### Arguments

| | |
|---|---|
| `C` | a ictmc class object |
| `i` | initial state at time t |
| `t` | initial time t. Default value = 0 |
| `s` | final time |
| `error` | error rate. Default value = 0.001 |
| `useRCpp` | logical whether to use RCpp implementation; by default TRUE |

### Author(s)

Vandit Jain

### References

Imprecise Continuous-Time Markov Chains, Thomas Krak et al., 2016

### Examples

```
states <- c("n","y")
Q <- matrix(c(-1,1,1,-1),nrow = 2,byrow = TRUE,dimnames = list(states,states))
range <- matrix(c(1/52,3/52,1/2,2),nrow = 2,byrow = 2)
name <- "testictmc"
ictmc <- new("ictmc",states = states,Q = Q,range = range,name = name)
impreciseProbabilityatT(ictmc,2,0,1,10^-3,TRUE)
```

`inferHyperparam` *Function to infer the hyperparameters for Bayesian inference from an a priori matrix or a data set*

## Description

Since the Bayesian inference approach implemented in the package is based on conjugate priors, hyperparameters must be provided to model the prior probability distribution of the chain parameters. The hyperparameters are inferred from a given a priori matrix under the assumption that the matrix provided corresponds to the mean (expected) values of the chain parameters. A scaling factor vector must be provided too. Alternatively, the hyperparameters can be inferred from a data set.

## Usage

```
inferHyperparam(transMatr = matrix(), scale = numeric(), data = character())
```

## Arguments

| | |
|---|---|
| `transMatr` | A valid transition matrix, with dimension names. |
| `scale` | A vector of scaling factors, each element corresponds to the row names of the provided transition matrix transMatr, in the same order. |
| `data` | A data set from which the hyperparameters are inferred. |

## Details

transMatr and scale need not be provided if data is provided.

## Value

Returns the hyperparameter matrix in a list.

## Note

The hyperparameter matrix returned is such that the row and column names are sorted alphanumerically, and the elements in the matrix are correspondingly permuted.

## Author(s)

Sai Bhargav Yalamanchi, Giorgio Spedicato

## References

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

## See Also

markovchainFit, predictiveDistribution

## Examples

```
data(rain, package = "markovchain")
inferHyperparam(data = rain$rain)

weatherStates <- c("sunny", "cloudy", "rain")
weatherMatrix <- matrix(data = c(0.7, 0.2, 0.1,
                                 0.3, 0.4, 0.3,
                                 0.2, 0.4, 0.4),
                        byrow = TRUE, nrow = 3,
                        dimnames = list(weatherStates, weatherStates))
inferHyperparam(transMatr = weatherMatrix, scale = c(10, 10, 10))
```

---

# is.accessible *Verify if a state j is reachable from state i.*

---

## Description

This function verifies if a state is reachable from another, i.e., if there exists a path that leads to state j leaving from state i with positive probability

## Usage

```
is.accessible(object, from, to)
```

## Arguments

| | |
|---|---|
| object | A markovchain object. |
| from | The name of state "i" (beginning state). |
| to | The name of state "j" (ending state). |

## Details

It wraps an internal function named reachabilityMatrix.

## Value

A boolean value.

## Author(s)

Giorgio Spedicato, Ignacio Cordón

## References

James Montgomery, University of Madison

## See Also

`is.irreducible`

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames,
               transitionMatrix = matrix(c(0.2, 0.5, 0.3,
                                           0,   1,   0,
                                         0.1, 0.8, 0.1), nrow = 3, byrow = TRUE,
                                        dimnames = list(statesNames, statesNames)
                                       )
               )
is.accessible(markovB, "a", "c")
```

---

`is.CTMCirreducible` *Check if CTMC is irreducible*

---

## Description

This function verifies whether a CTMC object is irreducible

## Usage

```
is.CTMCirreducible(ctmc)
```

## Arguments

| | |
|---|---|
| `ctmc` | a ctmc-class object |

## Value

a boolean value as described above.

## Author(s)

Vandit Jain

## References

Continuous-Time Markov Chains, Karl Sigman, Columbia University

## Examples

```
energyStates <- c("sigma", "sigma_star")
byRow <- TRUE
gen <- matrix(data = c(-3, 3,
                       1, -1), nrow = 2,
              byrow = byRow, dimnames = list(energyStates, energyStates))
molecularCTMC <- new("ctmc", states = energyStates,
                     byrow = byRow, generator = gen,
                     name = "Molecular Transition Model")
is.CTMCirreducible(molecularCTMC)
```

---

`is.irreducible` *Function to check if a Markov chain is irreducible (i.e. ergodic)*

---

## Description

This function verifies whether a `markovchain` object transition matrix is composed by only one communicating class.

## Usage

```
is.irreducible(object)
```

## Arguments

| | |
|---|---|
| `object` | A `markovchain` object |

## Details

It is based on `.communicatingClasses` internal function.

## Value

A boolean values.

## Author(s)

Giorgio Spedicato

## References

Feres, Matlab listings for Markov Chains.

## See Also

`summary`

## Examples

```
statesNames <- c("a", "b")
mcA <- new("markovchain", transitionMatrix = matrix(c(0.7,0.3,0.1,0.9),
                                                    byrow = TRUE, nrow = 2,
                                                    dimnames = list(statesNames, statesNames)
           ))
is.irreducible(mcA)
```

---

`is.regular` *Check if a DTMC is regular*

---

## Description

Function to check wether a DTCM is regular

## Usage

```
is.regular(object)
```

## Arguments

| | |
|---|---|
| `object` | a markovchain object |

## Details

A Markov chain is regular if some of the powers of its matrix has all elements strictly positive

## Value

A boolean value

## Author(s)

Ignacio Cordón

## References

Matrix Analysis. Roger A.Horn, Charles R.Johnson. 2nd edition. Corollary 8.5.8, Theorem 8.5.9

## See Also

`is.irreducible`

### Examples

```
P <- matrix(c(0.5,  0.25, 0.25,
              0.5,     0, 0.5,
              0.25, 0.25, 0.5), nrow = 3)
colnames(P) <- rownames(P) <- c("R","N","S")
ciao <- as(P, "markovchain")
is.regular(ciao)
```

---

## is.TimeReversible *checks if ctmc object is time reversible*

---

### Description

The function returns checks if provided function is time reversible

### Usage

```
is.TimeReversible(ctmc)
```

### Arguments

| | |
|---|---|
| ctmc | a ctmc-class object |

### Value

Returns a boolean value stating whether ctmc object is time reversible

a boolean value as described above

### Author(s)

Vandit Jain

### References

INTRODUCTION TO STOCHASTIC PROCESSES WITH R, ROBERT P. DOBROW, Wiley

### Examples

```
energyStates <- c("sigma", "sigma_star")
byRow <- TRUE
gen <- matrix(data = c(-3, 3,
                       1, -1), nrow = 2,
              byrow = byRow, dimnames = list(energyStates, energyStates))
molecularCTMC <- new("ctmc", states = energyStates,
                     byrow = byRow, generator = gen,
                     name = "Molecular Transition Model")
is.TimeReversible(molecularCTMC)
```

## kullback *Example from Kullback and Kupperman Tests for Contingency Tables*

### Description

A list of two matrices representing raw transitions between two states

### Usage

```
data(kullback)
```

### Format

A list containing two 6x6 non - negative integer matrices

## markovchain-class *Markov Chain class*

### Description

The S4 class that describes `markovchain` objects.

### Arguments

| | |
|---|---|
| `states` | Name of the states. Must be the same of `colnames` and `rownames` of the transition matrix |
| `byrow` | TRUE or FALSE indicating whether the supplied matrix is either stochastic by rows or by columns |
| `transitionMatrix` | Square transition matrix |
| `name` | Optional character name of the Markov chain |

### Creation of objects

Objects can be created by calls of the form `new("markovchain", states, byrow, transitionMatrix, ...)`.

### Methods

* `signature(e1 = "markovchain", e2 = "markovchain")`: multiply two `markovchain` objects
* `signature(e1 = "markovchain", e2 = "matrix")`: markovchain by matrix multiplication
* `signature(e1 = "markovchain", e2 = "numeric")`: markovchain by numeric vector multiplication
* `signature(e1 = "matrix", e2 = "markovchain")`: matrix by markov chain

* signature(e1 = "numeric", e2 = "markovchain"): numeric vector by markovchain multiplication

[ signature(x = "markovchain", i = "ANY", j = "ANY", drop = "ANY"): ...

^ signature(e1 = "markovchain", e2 = "numeric"): power of a markovchain object

== signature(e1 = "markovchain", e2 = "markovchain"): equality of two markovchain object

!= signature(e1 = "markovchain", e2 = "markovchain"): non-equality of two markovchain object

**absorbingStates** signature(object = "markovchain"): method to get absorbing states

**canonicForm** signature(object = "markovchain"): return a markovchain object into canonic form

**coerce** signature(from = "markovchain", to = "data.frame"): coerce method from markovchain to data.frame

**conditionalDistribution** signature(object = "markovchain"): returns the conditional probability of subsequent states given a state

**coerce** signature(from = "data.frame", to = "markovchain"): coerce method from data.frame to markovchain

**coerce** signature(from = "table", to = "markovchain"): coerce method from table to markovchain

**coerce** signature(from = "msm", to = "markovchain"): coerce method from msm to markovchain

**coerce** signature(from = "msm.est", to = "markovchain"): coerce method from msm.est (but only from a Probability Matrix) to markovchain

**coerce** signature(from = "etm", to = "markovchain"): coerce method from etm to markovchain

**coerce** signature(from = "sparseMatrix", to = "markovchain"): coerce method from sparseMatrix to markovchain

**coerce** signature(from = "markovchain", to = "igraph"): coercing to igraph objects

**coerce** signature(from = "markovchain", to = "matrix"): coercing to matrix objects

**coerce** signature(from = "markovchain", to = "sparseMatrix"): coercing to sparseMatrix objects

**coerce** signature(from = "matrix", to = "markovchain"): coercing to markovchain objects from matrix one

**dim** signature(x = "markovchain"): method to get the size

**names** signature(x = "markovchain"): method to get the names of states

**names<-** signature(x = "markovchain", value = "character"): method to set the names of states

**initialize** signature(.Object = "markovchain"): initialize method

**plot** signature(x = "markovchain", y = "missing"): plot method for markovchain objects

**predict** signature(object = "markovchain"): predict method

**print** signature(x = "markovchain"): print method.

**show** signature(object = "markovchain"): show method.

**sort** signature(x = "markovchain", decreasing=FALSE): sorting the transition matrix.

**states** `signature(object = "markovchain")`: returns the names of states (as `names`.

**steadyStates** `signature(object = "markovchain")`: method to get the steady vector.

**summary** `signature(object = "markovchain")`: method to summarize structure of the markov chain

**transientStates** `signature(object = "markovchain")`: method to get the transient states.

**t** `signature(x = "markovchain")`: transpose matrix

**transitionProbability** `signature(object = "markovchain")`: transition probability

## Note

1. `markovchain` object are backed by S4 Classes.
2. Validation method is used to assess whether either columns or rows totals to one. Rounding is used up to `.Machine$double.eps * 100`. If state names are not properly defined for a probability `matrix`, coercing to `markovchain` object leads to overriding states name with artificial "s1", "s2", ... sequence. In addition, operator overloading has been applied for $+, *, ^, ==, !=$ operators.

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

`markovchainSequence`,`markovchainFit`

## Examples

```
#show markovchain definition
showClass("markovchain")
#create a simple Markov chain
transMatr<-matrix(c(0.4,0.6,.3,.7),nrow=2,byrow=TRUE)
simpleMc<-new("markovchain", states=c("a","b"),
              transitionMatrix=transMatr,
              name="simpleMc")
#power
simpleMc^4
#some methods
steadyStates(simpleMc)
absorbingStates(simpleMc)
simpleMc[2,1]
t(simpleMc)
is.irreducible(simpleMc)
#conditional distributions
conditionalDistribution(simpleMc, "b")
#example for predict method
```

```
sequence<-c("a", "b", "a", "a", "a", "a", "b", "a", "b", "a", "b", "a", "a", "b", "b", "b", "a")
mcFit<-markovchainFit(data=sequence)
predict(mcFit$estimate, newdata="b",n.ahead=3)
#direct conversion
myMc<-as(transMatr, "markovchain")

#example of summary
summary(simpleMc)
## Not run: plot(simpleMc)
```

---

`markovchainList-class` *Non homogeneus discrete time Markov Chains class*

---

## Description

A class to handle non homogeneous discrete Markov chains

## Arguments

| | |
|---|---|
| `markovchains` | Object of class `"list"`: a list of markovchains |
| `name` | Object of class `"character"`: optional name of the class |

## Objects from the Class

A `markovchainlist` is a list of `markovchain` objects. They can be used to model non homogeneous discrete time Markov Chains, when transition probabilities (and possible states) change by time.

## Methods

**[[** `signature(x = "markovchainList")`: extract the i-th `markovchain`

**dim** `signature(x = "markovchainList")`: number of `markovchain` underlying the matrix

**predict** `signature(object = "markovchainList")`: predict from a `markovchainList`

**print** `signature(x = "markovchainList")`: prints the list of markovchains

**show** `signature(object = "markovchainList")`: same as `print`

## Note

The class consists in a list of `markovchain` objects. It is aimed at working with non homogeneous Markov chains.

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchain

## Examples

```
showClass("markovchainList")
#define a markovchainList
statesNames=c("a","b")

mcA<-new("markovchain",name="MCA",
         transitionMatrix=matrix(c(0.7,0.3,0.1,0.9),
                         byrow=TRUE, nrow=2,
                         dimnames=list(statesNames,statesNames))
        )

mcB<-new("markovchain", states=c("a","b","c"), name="MCB",
         transitionMatrix=matrix(c(0.2,0.5,0.3,0,1,0,0.1,0.8,0.1),
         nrow=3, byrow=TRUE))

mcC<-new("markovchain", states=c("a","b","c","d"), name="MCC",
         transitionMatrix=matrix(c(0.25,0.75,0,0,0.4,0.6,
                                   0,0,0,0,0.1,0.9,0,0,0.7,0.3),
                                 nrow=4, byrow=TRUE)
)
mcList<-new("markovchainList",markovchains=list(mcA, mcB, mcC),
           name="Non - homogeneous Markov Chain")
```

---

# markovchainListFit *markovchainListFit*

---

## Description

Given a data frame or a matrix (rows are observations, by cols the temporal sequence), it fits a non - homogeneous discrete time markov chain process (storing row). In particular a markovchainList of size = ncol - 1 is obtained estimating transitions from the n samples given by consecutive column pairs.

## Usage

```
markovchainListFit(data, byrow = TRUE, laplacian = 0, name)
```

## Arguments

| | |
|---|---|
| data | Either a matrix or a data.frame or a list object. |
| byrow | Indicates whether distinc stochastic processes trajectiories are shown in distinct rows. |
| laplacian | Laplacian correction (default 0). |
| name | Optional name. |

### Details

If data contains NAs then the transitions containing NA will be ignored.

### Value

A list containing two slots: estimate (the estimate) name

### Examples

```
# using holson dataset
data(holson)
# fitting a single markovchain
singleMc <- markovchainFit(data = holson[,2:12])
# fitting a markovchainList
mclistFit <- markovchainListFit(data = holson[, 2:12], name = "holsonMcList")
```

---

## markovchainSequence *Function to generate a sequence of states from homogeneous Markov chains.*

---

### Description

Provided any markovchain object, it returns a sequence of states coming from the underlying stationary distribution.

### Usage

```
markovchainSequence(
  n,
  markovchain,
  t0 = sample(markovchain@states, 1),
  include.t0 = FALSE,
  useRCpp = TRUE
)
```

### Arguments

| | |
|---|---|
| n | Sample size |
| markovchain | markovchain object |
| t0 | The initial state |
| include.t0 | Specify if the initial state shall be used |
| useRCpp | Boolean. Should RCpp fast implementation being used? Default is yes. |

### Details

A sequence of size n is sampled.

## Value

A Character Vector

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchainFit

## Examples

```
# define the markovchain object
statesNames <- c("a", "b", "c")
mcB <- new("markovchain", states = statesNames,
   transitionMatrix = matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1),
   nrow = 3, byrow = TRUE, dimnames = list(statesNames, statesNames)))

# show the sequence
outs <- markovchainSequence(n = 100, markovchain = mcB, t0 = "a")
```

---

# meanAbsorptionTime *Mean absorption time*

---

## Description

Computes the expected number of steps to go from any of the transient states to any of the recurrent states. The Markov chain should have at least one transient state for this method to work

## Usage

```
meanAbsorptionTime(object)
```

## Arguments

| | |
|---|---|
| object | the markovchain object |

## Value

A named vector with the expected number of steps to go from a transient state to any of the recurrent ones

### Author(s)

Ignacio Cordón

### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

### Examples

```
m <- matrix(c(1/2, 1/2, 0,
              1/2, 1/2, 0,
                0, 1/2, 1/2), ncol = 3, byrow = TRUE)
mc <- new("markovchain", states = letters[1:3], transitionMatrix = m)
times <- meanAbsorptionTime(mc)
```

---

## meanFirstPassageTime *Mean First Passage Time for irreducible Markov chains*

---

### Description

Given an irreducible (ergodic) markovchain object, this function calculates the expected number of steps to reach other states

### Usage

```
meanFirstPassageTime(object, destination)
```

### Arguments

| | |
|---|---|
| `object` | the markovchain object |
| `destination` | a character vector representing the states respect to which we want to compute the mean first passage time. Empty by default |

### Details

For an ergodic Markov chain it computes:

- If destination is empty, the average first time (in steps) that takes the Markov chain to go from initial state i to j. (i, j) represents that value in case the Markov chain is given row-wise, (j, i) in case it is given col-wise.
- If destination is not empty, the average time it takes us from the remaining states to reach the states in `destination`

### Value

a Matrix of the same size with the average first passage times if destination is empty, a vector if destination is not

### Author(s)

Toni Giorgino, Ignacio Cordón

### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

### Examples

```
m <- matrix(1 / 10 * c(6,3,1,
                       2,3,5,
                       4,1,5), ncol = 3, byrow = TRUE)
mc <- new("markovchain", states = c("s","c","r"), transitionMatrix = m)
meanFirstPassageTime(mc, "r")


# Grinstead and Snell's "Oz weather" worked out example
mOz <- matrix(c(2,1,1,
                2,0,2,
                1,1,2)/4, ncol = 3, byrow = TRUE)

mcOz <- new("markovchain", states = c("s", "c", "r"), transitionMatrix = mOz)
meanFirstPassageTime(mcOz)
```

---

## meanNumVisits *Mean num of visits for markovchain, starting at each state*

---

### Description

Given a markovchain object, this function calculates a matrix where the element (i, j) represents the expect number of visits to the state j if the chain starts at i (in a Markov chain by columns it would be the element (j, i) instead)

### Usage

```
meanNumVisits(object)
```

### Arguments

| | |
|---|---|
| object | the markovchain-class object |

### Value

a matrix with the expect number of visits to each state

### Author(s)

Ignacio Cordón

## References

R. Vélez, T. Prieto, Procesos Estocásticos, Librería UNED, 2013

## Examples

```
M <- markovchain:::zeros(5)
M[1,1] <- M[5,5] <- 1
M[2,1] <- M[2,3] <- 1/2
M[3,2] <- M[3,4] <- 1/2
M[4,2] <- M[4,5] <- 1/2

mc <- new("markovchain", transitionMatrix = M)
meanNumVisits(mc)
```

---

# meanRecurrenceTime *Mean recurrence time*

---

## Description

Computes the expected time to return to a recurrent state in case the Markov chain starts there

## Usage

```
meanRecurrenceTime(object)
```

## Arguments

| | |
|---|---|
| object | the markovchain object |

## Value

For a Markov chain it outputs is a named vector with the expected time to first return to a state when the chain starts there. States present in the vector are only the recurrent ones. If the matrix is ergodic (i.e. irreducible), then all states are present in the output and order is the same as states order for the Markov chain

## Author(s)

Ignacio Cordón

## References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

## Examples

```
m <- matrix(1 / 10 * c(6,3,1,
                       2,3,5,
                       4,1,5), ncol = 3, byrow = TRUE)
mc <- new("markovchain", states = c("s","c","r"), transitionMatrix = m)
meanRecurrenceTime(mc)
```

---

`multinomialConfidenceIntervals` *A function to compute multinomial confidence intervals of DTMC*

---

## Description

Return estimated transition matrix assuming a Multinomial Distribution

## Usage

```
multinomialConfidenceIntervals(
  transitionMatrix,
  countsTransitionMatrix,
  confidencelevel = 0.95
)
```

## Arguments

`transitionMatrix`
: An estimated transition matrix.

`countsTransitionMatrix`
: Empirical (conts) transition matrix, on which the `transitionMatrix` was performed.

`confidencelevel`
: confidence interval level.

## Value

Two matrices containing the confidence intervals.

## References

Constructing two-sided simultaneous confidence intervals for multinomial proportions for small counts in a large number of cells. Journal of Statistical Software 5(6) (2000)

## See Also

`markovchainFit`

## Examples

```
seq<-c("a", "b", "a", "a", "a", "a", "b", "a", "b", "a", "b", "a", "a", "b", "b", "b", "a")
mcfit<-markovchainFit(data=seq,byrow=TRUE)
seqmat<-createSequenceMatrix(seq)
multinomialConfidenceIntervals(mcfit$estimate@transitionMatrix, seqmat, 0.95)
```

---

name *Method to retrieve name of markovchain object*

---

## Description

This method returns the name of a markovchain object

## Usage

```
name(object)

## S4 method for signature 'markovchain'
name(object)
```

## Arguments

| | |
|---|---|
| object | A markovchain object |

## Author(s)

Giorgio Spedicato, Deepak Yadav

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
               matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1), nrow = 3,
               byrow = TRUE, dimnames=list(statesNames,statesNames)),
               name = "A markovchain Object"
)
name(markovB)
```

## name<- *Method to set name of markovchain object*

### Description

This method modifies the existing name of markovchain object

### Usage

```
name(object) <- value

## S4 replacement method for signature 'markovchain'
name(object) <- value
```

### Arguments

| | |
|---|---|
| object | A markovchain object |
| value | New name of markovchain object |

### Author(s)

Giorgio Spedicato, Deepak Yadav

### Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
                matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1), nrow = 3,
                byrow = TRUE, dimnames=list(statesNames,statesNames)),
                name = "A markovchain Object"
)
name(markovB) <- "dangerous mc"
```

## names,markovchain-method *Returns the states for a Markov chain object*

### Description

Returns the states for a Markov chain object

### Usage

```
## S4 method for signature 'markovchain'
names(x)
```

### Arguments

| | |
|---|---|
| x | object we want to return states for |

---

`noofVisitsDist` *return a joint pdf of the number of visits to the various states of the DTMC*

---

### Description

This function would return a joint pdf of the number of visits to the various states of the DTMC during the first N steps.

### Usage

```
noofVisitsDist(markovchain,N,state)
```

### Arguments

| | |
|---|---|
| markovchain | a markovchain-class object |
| N | no of steps |
| state | the initial state |

### Details

This function would return a joint pdf of the number of visits to the various states of the DTMC during the first N steps.

### Value

a numeric vector depicting the above described probability density function.

### Author(s)

Vandit Jain

### Examples

```
transMatr<-matrix(c(0.4,0.6,.3,.7),nrow=2,byrow=TRUE)
simpleMc<-new("markovchain", states=c("a","b"),
              transitionMatrix=transMatr,
              name="simpleMc")
noofVisitsDist(simpleMc,5,"a")
```

## ones *Returns an Identity matrix*

### Description

Returns an Identity matrix

### Usage

```
ones(n)
```

### Arguments

| | |
|---|---|
| n | size of the matrix |

### Value

a identity matrix

## period *Various function to perform structural analysis of DTMC*

### Description

These functions return absorbing and transient states of the markovchain objects.

### Usage

```
period(object)

communicatingClasses(object)

recurrentClasses(object)

transientClasses(object)

transientStates(object)

recurrentStates(object)

absorbingStates(object)

canonicForm(object)
```

## Arguments

| | |
|---|---|
| object | A markovchain object. |

## Value

period returns a integer number corresponding to the periodicity of the Markov chain (if it is irreducible)

absorbingStates returns a character vector with the names of the absorbing states in the Markov chain

communicatingClasses returns a list in which each slot contains the names of the states that are in that communicating class

recurrentClasses analogously to communicatingClasses, but with recurrent classes

transientClasses analogously to communicatingClasses, but with transient classes

transientStates returns a character vector with all the transient states for the Markov chain

recurrentStates returns a character vector with all the recurrent states for the Markov chain

canonicForm returns the Markov chain reordered by a permutation of states so that we have blocks submatrices for each of the recurrent classes and a collection of rows in the end for the transient states

## Author(s)

Giorgio Alfredo Spedicato, Ignacio Cordón

## References

Feres, Matlab listing for markov chain.

## See Also

markovchain

## Examples

```
statesNames <- c("a", "b", "c")
mc <- new("markovchain", states = statesNames, transitionMatrix =
          matrix(c(0.2, 0.5, 0.3,
                   0,   1,   0,
                   0.1, 0.8, 0.1), nrow = 3, byrow = TRUE,
                 dimnames = list(statesNames, statesNames))
         )

communicatingClasses(mc)
recurrentClasses(mc)
recurrentClasses(mc)
absorbingStates(mc)
transientStates(mc)
recurrentStates(mc)
canonicForm(mc)
```

```
# periodicity analysis
A <- matrix(c(0, 1, 0, 0, 0.5, 0, 0.5, 0, 0, 0.5, 0, 0.5, 0, 0, 1, 0),
            nrow = 4, ncol = 4, byrow = TRUE)
mcA <- new("markovchain", states = c("a", "b", "c", "d"),
          transitionMatrix = A,
          name = "A")

is.irreducible(mcA) #true
period(mcA) #2

# periodicity analysis
B <- matrix(c(0, 0, 1/2, 1/4, 1/4, 0, 0,
                   0, 0, 1/3, 0, 2/3, 0, 0,
                   0, 0, 0, 0, 0, 1/3, 2/3,
                   0, 0, 0, 0, 0, 1/2, 1/2,
                   0, 0, 0, 0, 0, 3/4, 1/4,
                   1/2, 1/2, 0, 0, 0, 0, 0,
                   1/4, 3/4, 0, 0, 0, 0, 0), byrow = TRUE, ncol = 7)
mcB <- new("markovchain", transitionMatrix = B)
period(mcB)
```

---

predictHommc *Simulate a higher order multivariate markovchain*

---

## Description

This function provides a prediction of states for a higher order multivariate markovchain object

## Usage

```
predictHommc(hommc,t,init)
```

## Arguments

| | |
|---|---|
| hommc | a hommc-class object |
| t | no of iterations to predict |
| init | matrix of previous states size of which depends on hommc |

## Details

The user is required to provide a matrix of giving n previous coressponding every categorical sequence. Dimensions of the init are s X n, where s is number of categorical sequences and n is order of the homc.

## Value

The function returns a matrix of size s X t displaying t predicted states in each row coressponding to every categorical sequence.

### Author(s)

Vandit Jain

---

`predictiveDistribution` *predictiveDistribution*

---

### Description

The function computes the probability of observing a new data set, given a data set

### Usage

```
predictiveDistribution(stringchar, newData, hyperparam = matrix())
```

### Arguments

| | |
|---|---|
| `stringchar` | This is the data using which the Bayesian inference is performed. |
| `newData` | This is the data whose predictive probability is computed. |
| `hyperparam` | This determines the shape of the prior distribution of the parameters. If none is provided, default value of 1 is assigned to each parameter. This must be of size kxk where k is the number of states in the chain and the values should typically be non-negative integers. |

### Details

The underlying method is Bayesian inference. The probability is computed by averaging the likelihood of the new data with respect to the posterior. Since the method assumes conjugate priors, the result can be represented in a closed form (see the vignette for more details), which is what is returned.

### Value

The log of the probability is returned.

### Author(s)

Sai Bhargav Yalamanchi

### References

Inferring Markov Chains: Bayesian Estimation, Model Comparison, Entropy Rate, and Out-of-Class Modeling, Christopher C. Strelioff, James P. Crutchfield, Alfred Hubler, Santa Fe Institute

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

## See Also

markovchainFit

## Examples

```
sequence<- c("a", "b", "a", "a", "a", "a", "b", "a", "b", "a", "b", "a", "a",
             "b", "b", "b", "a")
hyperMatrix<-matrix(c(1, 2, 1, 4), nrow = 2,dimnames=list(c("a","b"),c("a","b")))
predProb <- predictiveDistribution(sequence[1:10], sequence[11:17], hyperparam =hyperMatrix )
hyperMatrix2<-hyperMatrix[c(2,1),c(2,1)]
predProb2 <- predictiveDistribution(sequence[1:10], sequence[11:17], hyperparam =hyperMatrix2 )
predProb2==predProb
```

---

# preproglucacon *Preprogluccacon DNA protein bases sequences*

---

## Description

Sequence of bases for preproglucacon DNA protein

## Usage

```
data(preproglucacon)
```

## Format

A data frame with 1572 observations on the following 2 variables.

`V1` a numeric vector, showing original coding

`preproglucacon` a character vector, showing initial of DNA bases (Adenine, Cytosine, Guanine, Thymine)

## Source

Avery Henderson

## References

Averuy Henderson, Fitting markov chain models on discrete time series such as DNA sequences

## Examples

```
data(preproglucacon)
preproglucaconMc<-markovchainFit(data=preproglucacon$preproglucacon)
```

priorDistribution *priorDistribution*

## Description

Function to evaluate the prior probability of a transition matrix. It is based on conjugate priors and therefore a Dirichlet distribution is used to model the transitions of each state.

## Usage

```
priorDistribution(transMatr, hyperparam = matrix())
```

## Arguments

| | |
|---|---|
| transMatr | The transition matrix whose probability is the parameter of interest. |
| hyperparam | The hyperparam matrix (optional). If not provided, a default value of 1 is assumed for each and therefore the resulting probability distribution is uniform. |

## Details

The states (dimnames) of the transition matrix and the hyperparam may be in any order.

## Value

The log of the probabilities for each state is returned in a numeric vector. Each number in the vector represents the probability (log) of having a probability transition vector as specified in corresponding the row of the transition matrix.

## Note

This function can be used in conjunction with inferHyperparam. For example, if the user has a prior data set and a prior transition matrix, he can infer the hyperparameters using inferHyperparam and then compute the probability of their prior matrix using the inferred hyperparameters with priorDistribution.

## Author(s)

Sai Bhargav Yalamanchi, Giorgio Spedicato

## References

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

## See Also

predictiveDistribution, inferHyperparam

## Examples

```
priorDistribution(matrix(c(0.5, 0.5, 0.5, 0.5),
                  nrow = 2,
                  dimnames = list(c("a", "b"), c("a", "b"))),
                  matrix(c(2, 2, 2, 2),
                  nrow = 2,
                  dimnames = list(c("a", "b"), c("a", "b"))))
```

---

`probabilityatT` *Calculating probability from a ctmc object*

---

## Description

This function returns the probability of every state at time t under different conditions

## Usage

```
probabilityatT(C,t,x0,useRCpp)
```

## Arguments

| | |
|---|---|
| `C` | A CTMC S4 object |
| `t` | final time t |
| `x0` | initial state |
| `useRCpp` | logical whether to use RCpp implementation |

## Details

The initial state is not mandatory, In case it is not provided, function returns a matrix of transition function at time `t` else it returns vector of probaabilities of transition to different states if initial state was `x0`

## Value

returns a vector or a matrix in case `x0` is provided or not respectively.

## Author(s)

Vandit Jain

## References

INTRODUCTION TO STOCHASTIC PROCESSES WITH R, ROBERT P. DOBROW, Wiley

## Examples

```
states <- c("a","b","c","d")
byRow <- TRUE
gen <- matrix(data = c(-1, 1/2, 1/2, 0, 1/4, -1/2, 0, 1/4, 1/6, 0, -1/3, 1/6, 0, 0, 0, 0),
nrow = 4,byrow = byRow, dimnames = list(states,states))
ctmc <- new("ctmc",states = states, byrow = byRow, generator = gen, name = "testctmc")
probabilityatT(ctmc,1,useRCpp = TRUE)
```

---

# rain *Alofi island daily rainfall*

---

## Description

Rainfall measured in Alofi Island

## Usage

```
data(rain)
```

## Format

A data frame with 1096 observations on the following 2 variables.

`V1` a numeric vector, showing original coding

`rain` a character vector, showing daily rainfall millilitres brackets

## Source

Avery Henderson

## References

Avery Henderson, Fitting markov chain models on discrete time series such as DNA sequences

## Examples

```
data(rain)
rainMc<-markovchainFit(data=rain$rain)
```

rctmc *rctmc*

## Description

The function generates random CTMC transitions as per the provided generator matrix.

## Usage

```
rctmc(n, ctmc, initDist = numeric(), T = 0, include.T0 = TRUE,
  out.type = "list")
```

## Arguments

| | |
|---|---|
| n | The number of samples to generate. |
| ctmc | The CTMC S4 object. |
| initDist | The initial distribution of states. |
| T | The time up to which the simulation runs (all transitions after time T are not returned). |
| include.T0 | Flag to determine if start state is to be included. |
| out.type | "list" or "df" |

## Details

In order to use the T0 argument, set n to Inf.

## Value

Based on out.type, a list or a data frame is returned. The returned list has two elements - a character vector (states) and a numeric vector (indicating time of transitions). The data frame is similarly structured.

## Author(s)

Sai Bhargav Yalamanchi

## References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison

## See Also

generatorToTransitionMatrix,ctmc-class

## Examples

```
energyStates <- c("sigma", "sigma_star")
byRow <- TRUE
gen <- matrix(data = c(-3, 3, 1, -1), nrow = 2,
              byrow = byRow, dimnames = list(energyStates, energyStates))
molecularCTMC <- new("ctmc", states = energyStates,
                     byrow = byRow, generator = gen,
                     name = "Molecular Transition Model")

statesDist <- c(0.8, 0.2)
rctmc(n = Inf, ctmc = molecularCTMC, T = 1)
rctmc(n = 5, ctmc = molecularCTMC, initDist = statesDist, include.T0 = FALSE)
```

---

`rmarkovchain` *Function to generate a sequence of states from homogeneous or non-homogeneous Markov chains.*

---

## Description

Provided any `markovchain` or `markovchainList` objects, it returns a sequence of states coming from the underlying stationary distribution.

## Usage

```
rmarkovchain(
  n,
  object,
  what = "data.frame",
  useRCpp = TRUE,
  parallel = FALSE,
  num.cores = NULL,
  ...
)
```

## Arguments

| | |
|---|---|
| `n` | Sample size |
| `object` | Either a `markovchain` or a `markovchainList` object |
| `what` | It specifies whether either a `data.frame` or a `matrix` (each rows represent a simulation) or a `list` is returned. |
| `useRCpp` | Boolean. Should RCpp fast implementation being used? Default is yes. |
| `parallel` | Boolean. Should parallel implementation being used? Default is yes. |
| `num.cores` | Number of Cores to be used |
| `...` | additional parameters passed to the internal sampler |

## Details

When a homogeneous process is assumed (`markovchain` object) a sequence is sampled of size n. When a non - homogeneous process is assumed, n samples are taken but the process is assumed to last from the begin to the end of the non-homogeneous markov process.

## Value

Character Vector, data.frame, list or matrix

## Note

Check the type of input

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

`markovchainFit`, `markovchainSequence`

## Examples

```
# define the markovchain object
statesNames <- c("a", "b", "c")
mcB <- new("markovchain", states = statesNames,
   transitionMatrix = matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1),
   nrow = 3, byrow = TRUE, dimnames = list(statesNames, statesNames)))

# show the sequence
outs <- rmarkovchain(n = 100, object = mcB, what = "list")


#define markovchainList object
statesNames <- c("a", "b", "c")
mcA <- new("markovchain", states = statesNames, transitionMatrix =
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mcB <- new("markovchain", states = statesNames, transitionMatrix =
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mcC <- new("markovchain", states = statesNames, transitionMatrix =
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mclist <- new("markovchainList", markovchains = list(mcA, mcB, mcC))

# show the list of sequence
```

```
rmarkovchain(100, mclist, "list")
```

---

`sales` *Sales Demand Sequences*

---

## Description

Sales demand sequences of five products (A, B, C, D, E). Each row corresponds to a sequence. First row corresponds to Sequence A, Second row to Sequence B and so on.

## Usage

```
data("sales")
```

## Format

An object of class `matrix` (inherits from `array`) with 269 rows and 5 columns.

## Details

The example can be used to fit High order multivariate markov chain.

## Examples

```
data("sales")
# fitHighOrderMultivarMC(seqMat = sales, order = 2, Norm = 2)
```

---

`show,hommc-method` *Function to display the details of hommc object*

---

## Description

This is a convenience function to display the slots of hommc object in proper format

## Usage

```
## S4 method for signature 'hommc'
show(object)
```

## Arguments

| | |
|---|---|
| `object` | An object of class hommc |

states *Defined states of a transition matrix*

## Description

This method returns the states of a transition matrix.

## Usage

```
states(object)

## S4 method for signature 'markovchain'
states(object)
```

## Arguments

| | |
|---|---|
| object | A discrete markovchain object |

## Value

The character vector corresponding to states slot.

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchain

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
                matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1), nrow = 3,
                byrow = TRUE, dimnames=list(statesNames,statesNames)),
                name = "A markovchain Object"
)
states(markovB)
names(markovB)
```

steadyStates *Stationary states of a* markovchain *object*

## Description

This method returns the stationary vector in matricial form of a markovchain object.

## Usage

```
steadyStates(object)
```

## Arguments

| | |
|---|---|
| object | A discrete markovchain object |

## Value

A matrix corresponding to the stationary states

## Note

The steady states are identified starting from which eigenvectors correspond to identity eigenvalues and then normalizing them to sum up to unity. When negative values are found in the matrix, the eigenvalues extraction is performed on the recurrent classes submatrix.

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchain

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
                matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1), nrow = 3,
                byrow = TRUE, dimnames=list(statesNames,statesNames)),
               name = "A markovchain Object"
)
steadyStates(markovB)
```

## tm_abs *Single Year Corporate Credit Rating Transitions*

### Description

Matrix of Standard and Poor's Global Corporate Rating Transition Frequencies 2000 (NR Removed)

### Usage

```
data(tm_abs)
```

### Format

The format is: num [1:8, 1:8] 17 2 0 0 0 0 0 0 1 455 ... - attr(*, "dimnames")=List of 2 ..$ : chr [1:8] "AAA" "AA" "A" "BBB" ... ..$ : chr [1:8] "AAA" "AA" "A" "BBB" ...

### References

European Securities and Markets Authority, 2016 https://cerep.esma.europa.eu/cerep-web/statistics/transitionMatrice.xhtml

### Examples

```
data(tm_abs)
```

## transition2Generator *Return the generator matrix for a corresponding transition matrix*

### Description

Calculate the generator matrix for a corresponding transition matrix

### Usage

```
transition2Generator(P, t = 1, method = "logarithm")
```

### Arguments

| | |
|---|---|
| P | transition matrix between time 0 and t |
| t | time of observation |
| method | "logarithm" returns the Matrix logarithm of the transition matrix |

### Value

A matrix that represent the generator of P

## See Also

rctmc

## Examples

```
mymatr <- matrix(c(.4, .6, .1, .9), nrow = 2, byrow = TRUE)
Q <- transition2Generator(P = mymatr)
expm::expm(Q)
```

---

# transitionProbability *Function to get the transition probabilities from initial to subsequent states.*

---

## Description

This is a convenience function to get transition probabilities.

## Usage

```
transitionProbability(object, t0, t1)

## S4 method for signature 'markovchain'
transitionProbability(object, t0, t1)
```

## Arguments

| | |
|---|---|
| object | A markovchain object. |
| t0 | Initial state. |
| t1 | Subsequent state. |

## Value

Numeric Vector

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

## See Also

markovchain

## Examples

```
statesNames <- c("a", "b", "c")
markovB <- new("markovchain", states = statesNames, transitionMatrix =
                matrix(c(0.2, 0.5, 0.3, 0, 1, 0, 0.1, 0.8, 0.1), nrow = 3,
                byrow = TRUE, dimnames=list(statesNames,statesNames)),
               name = "A markovchain Object"
)
transitionProbability(markovB,"b", "c")
```

---

# verifyMarkovProperty *Various functions to perform statistical inference of DTMC*

---

## Description

These functions verify the Markov property, assess the order and stationarity of the Markov chain.

This function tests whether an empirical transition matrix is statistically compatible with a theoretical one. It is a chi-square based test. In case a cell in the empirical transition matrix is >0 that is 0 in the theoretical transition matrix the null hypothesis is rejected.

Verifies that the s elements in the input list belongs to the same DTMC

## Usage

```
verifyMarkovProperty(sequence, verbose = TRUE)

assessOrder(sequence, verbose = TRUE)

assessStationarity(sequence, nblocks, verbose = TRUE)

verifyEmpiricalToTheoretical(data, object, verbose = TRUE)

verifyHomogeneity(inputList, verbose = TRUE)
```

## Arguments

| | |
|---|---|
| sequence | An empirical sequence. |
| verbose | Should test results be printed out? |
| nblocks | Number of blocks. |
| data | matrix, character or list to be converted in a raw transition matrix |
| object | a markovchain object |
| inputList | A list of items that can coerced to transition matrices |

## Value

Verification result

a list with following slots: statistic (the chi - square statistic), dof (degrees of freedom), and corresponding p-value. In case a cell in the empirical transition matrix is >0 that is 0 in the theoretical transition matrix the null hypothesis is rejected. In that case a p-value of 0 and statistic and dof of NA are returned.

a list of transition matrices?

## Author(s)

Tae Seung Kang, Giorgio Alfredo Spedicato

## References

Anderson and Goodman.

## See Also

markovchain

## Examples

```
sequence <- c("a", "b", "a", "a", "a", "a", "b", "a", "b",
              "a", "b", "a", "a", "b", "b", "b", "a")
mcFit <- markovchainFit(data = sequence, byrow = FALSE)
verifyMarkovProperty(sequence)
assessOrder(sequence)
assessStationarity(sequence, 1)



#Example taken from Kullback Kupperman Tests for Contingency Tables and Markov Chains

sequence<-c(0,1,2,2,1,0,0,0,0,0,0,1,2,2,2,1,0,0,1,0,0,0,0,0,0,1,1,
2,0,0,2,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,2,1,0,
0,2,1,0,0,0,0,0,0,1,1,1,2,2,0,0,2,1,1,1,1,2,1,1,1,1,1,1,1,1,0,2,
0,1,1,0,0,0,1,2,2,0,0,0,0,0,0,2,2,2,1,1,1,1,0,1,1,1,1,0,0,2,1,1,
0,0,0,0,0,2,2,1,1,1,1,1,2,1,2,0,0,0,1,2,2,2,0,0,0,1,1)

mc=matrix(c(5/8,1/4,1/8,1/4,1/2,1/4,1/4,3/8,3/8),byrow=TRUE, nrow=3)
rownames(mc)<-colnames(mc)<-0:2; theoreticalMc<-as(mc, "markovchain")

verifyEmpiricalToTheoretical(data=sequence,object=theoreticalMc)


data(kullback)
verifyHomogeneity(inputList=kullback,verbose=TRUE)
```

`zeros` *Matrix to create zeros*

## Description

Matrix to create zeros

## Usage

```
zeros(n)
```

## Arguments

| | |
|---|---|
| `n` | size of the matrix |

## Value

a square matrix of zeros

# Index